Brief Description
Introduction to the nonlinear dynamics of continuous-time and discrete-time systems. Routes to
chaos. Quantification of chaos. Basics of nonlinear time series analysis. Applications to chaos
synchronization, chaos-based communication, chaos control, and neuromorphic computing. TIA:
Systems and Controls.

Textbooks

0-521-01084-9 (optional).

S.H. Strogatz, Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and

Prerequisites
Math 2401 or 2411 or equivalent.
Math 2403 or 2413 or equivalent.
CEE/ISyE/MATH 3770 or ISyE 2027 or equivalent.

Instructor
Dr. Alexandre Locquet, Office 206.
Communication: Please send me messages using Canvas. ("Inbox" tab on your dashboard)

Lectures
TBA

Office hours
TBA
Assessment

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Attendance

Students who do not miss more than 2 lectures during the entire term get 5% for attendance. Students who miss more than 2 classes get 0% for attendance. Attendance Polling using Turning Point Technology and/or attendance sheets will be used to count the number of absences. Students need to install the TurningPoint app on a mobile device and bring their mobile device to class. As a Georgia Tech student, installation and use of the app is free. In the TurningPoint settings, you need to set your region to "North/South America" and not to Europe. Students who do not miss any class get a 1% attendance bonus.

Homework

Around 5 problems will be assigned. An assignment might be due on the week before final exams. Please box all answers. Homework should be submitted electronically on Canvas as a single, legible, pdf file. You are allowed to collaborate on homework with other students, but all work to be submitted should then be worked out and written up on your own. Of course, copying solutions from a Faculty solutions manual is cheating. A completion grade will be assigned. If a given homework 1) has been submitted on time and 2) every problem is answered, the student gets 2% credit. Homework turned in late loses 50%. Homework turned in two or more days late will not earn credit.

Final Exam

The final examination will be closed-book and notes. It will be 3-hour long and comprehensive. The use of a calculator will be allowed. Illness is an appropriate reason for missing the final examination, but you will need to produce a doctor’s note stating that you are not able to take the final.

Project

Students will choose a project related to a specific subject in nonlinear dynamics at the beginning of the term. Students are required to make a 5-minute presentation on January 30, a 10-minute presentation on February 27 and a final 15-minute presentation at the end of the term, as well as provide a 10-page report.

Examples of possible topics:

- Design of fuel efficient space missions (Lagrange points, Interplanetary Transport Network,...)
- Chaotic Mixing of microfluids
- Chaos synchronization of networks of dynamical systems (cluster formation, chimera states, etc...)
- Reservoir (neuromorphic) computing with nonlinear dynamical systems
- Heart nonlinear dynamics / Sudden death forecasting
- Nonlinear dynamics in neuron behavior / inter-neuron communication
- Rogue waves
- Chaos in electronic circuits
- Chaos in lasers and synchronization of chaotic lasers
- Pattern formation in spatially extended systems
- Stochastic Resonance
- Stability/instability of electric power grids
- Nonlinear dynamics in gene regulation
- Chaos control
- Embedding theorems (nonlinear time series analysis) and their applications

Students will typically make extensive use of the digital library: [http://www.library.gatech.edu/](http://www.library.gatech.edu/) to get started on the project.

**In-Lecture Polling**

Students may be asked to answer questions during some lectures, using the Turning Point app.

**Topical Outline**

1. Dynamics of Iterated Maps

   One- and Two-dimensional maps
   Limit sets: fixed points, periodic points, chaotic attractors
   Stability: sinks, sources, saddles, stable and unstable manifolds.
   Chaotic Orbits: sensitivity to initial conditions, Lyapunov exponents, fractals.

2. Dynamics of Continuous-Time Systems

   Linear and nonlinear systems. Poincaré-Bendixson theorem.
   Stability: characteristic values, Lyapunov functions, Floquet multipliers, Lyapunov exponents.
   Lab demonstrations: the Lorenz system, chaos in laser diodes.

3. Bifurcations

   Saddle-node, period-doubling, Hopf, and torus bifurcations.
   Bifurcation diagrams and routes to chaos: period-doubling cascade, quasi-periodicity, intermittency and crises.
   Lab demonstration: period-doubling cascade in the Lorenz system, bifurcations in the Chua circuit.

4. Quantifying chaos

   Lyapunov spectrum.
   Fractal dimensions.
Kolmogorov-Sinai Entropy.

5. Basics of nonlinear time series analysis

State reconstruction from data through delay embedding. Simple nonlinear prediction and noise reduction. Computation of the largest Lyapunov exponent from data.

6. Coupled chaotic systems and applications


Tentative Table of Contents

I. Basic Notions on Dynamical systems
   I.1 What is Chaos?
   I.2 Classification of Dynamical Systems.
      I.2.1 What is a Dynamical System?
      I.2.2 Classification

II. One-Dimensional Maps
   II.1 Introduction and Definitions
   II.2 Stability of Fixed Points
   II.3 Periodic Orbits
   II.4 Sensitive Dependence on Initial Conditions
   II.5 Itineraries and Symbolic Dynamics
   II.6 Bifurcations of Smooth One-Dimensional Maps
      II.6.1 Period-Doubling Bifurcation
      II.6.2 Saddle-Node Bifurcation
      II.6.3 The Transcritical Bifurcation

III. Two-Dimensional Maps
   III.1 Introduction and Definitions
   III.2 Stability of Fixed Points
   III.3 Linear Maps
   III.4 Nonlinear Maps
      III.4.1 Stability of Fixed and Periodic Points
      III.4.2 Stable and Unstable Manifolds of Saddle Points
      III.5 (Local) Bifurcations in 2D Maps

IV. Chaos
   IV.1 Introduction
   IV.2 Lyapunov Exponent of 1D Maps
   IV.3 Chaotic Orbits
   IV.4 Chaos in the Logistic Map G=4x(1-x)
      IV.4.1 Conjugacy
IV.5 Lyapunov Exponents in Higher Dimensions
IV.6 Chaotic Orbits in Higher Dimensions
IV.7 Numerical Calculation of Lyapunov Exponents
IV.8 Chaotic Attractors
IV.8.1 Limit Sets
IV.8.2 Dimension of Chaotic Attractors

V. Continuous-Time Systems
V.1 Introduction
V.2 Existence and Uniqueness
V.3 Linear 1D Differential Equations
V.4 Equilibrium and Stability
V.5 Nonlinear 1D Differential Equations
V.6 n-Dimensional Linear Differential Equations
V.7 Stability Criterion for Linear and Nonlinear Systems
V.8 Nonlinear Differential Equations in More than One Dimension
V.8.1 Properties of forward-limit sets
V.8.2 The Poincaré-Bendixson Theorem
V.9 Chaos in Differential Equations
V.9.1 Quasi-Periodic Signals
V.9.2 Lyapunov Exponents
V.9.3 Example of Chaotic Systems
V.10 Bifurcation of Equilibrium Points
V.10.1 Saddle-Node Bifurcation
V.10.2 Andronov-Hopf Bifurcation
V.11 Bifurcation of Limit Cycles
V.11.1 Saddle-Node Bifurcations of Limit Cycles
V.11.2 Period-Doubling Bifurcation of Limit Cycles
V.12 Global Bifurcations and Crisis
V.13 Famous Routes to Chaos
V.13.1 Period-Doubling Route to Chaos
V.13.2 Quasi-Periodic Route to Chaos
V.13.3 Intermittency Transition to a Chaotic Attractor
V.13.4 Transition to a Chaotic Attractor through a Crisis

VI. Experimental Time Series Analysis
VI.1 Introduction
VI.2 Embedding Theorems and Phase Space Reconstruction
VI.3 Examples: modeling, prediction, parameter identification, estimation of attractor dimension

VII Applications of Chaos Theory
VII.1 Chaos in Electronic Systems
VII.2 Chaos in Laser Systems
VII.3 Synchronization of Chaotic Systems and Chaos-Based Communication
VII.4 Random-Number Generation
VII.5 Neuromorphic Computing Exploiting Transient States in Dynamical Systems
VII.6 Compressive sensing exploiting the pseudo-randomness of a chaotic system

Student-Faculty Expectations Agreement

At Georgia Tech we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body.
See [http://www.catalog.gatech.edu/rules/22/](http://www.catalog.gatech.edu/rules/22/) for an articulation of some basic expectation that you can have of me and that I have of you. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, I encourage you to remain committed to the ideals of Georgia Tech while in this class.

**Honor Code**

Students are, of course, expected to abide by the [Georgia Tech Honor Code](http://www.honor.gatech.edu) and the “GTL student policies” document that you will find in the resources section of the Tsquare website for this course. Instances of academic misconduct will be viewed very seriously. For any questions involving Academic Honor Code issues, please consult me or visit [www.honor.gatech.edu](http://www.honor.gatech.edu).

**Student Feedback**

Anonymous feedback can be provided to the instructor using the link below:

[https://docs.google.com/forms/d/e/1FAIpQLSdPBLPaEyohxyQY-w6nBh-dznH4sPP52Mx6XZKwl3WLB6Obg/viewform?usp=sf_link](https://docs.google.com/forms/d/e/1FAIpQLSdPBLPaEyohxyQY-w6nBh-dznH4sPP52Mx6XZKwl3WLB6Obg/viewform?usp=sf_link)

You are also encouraged to fill in the course-instructor opinion survey (CIOS).