

ECE 8803 – Nonlinear Dynamics and Applications

Spring 2018

Georgia Tech Lorraine

Brief Description

Introduction to the nonlinear dynamics of continuous-time and discrete-time systems. Routes to chaos. Quantification of chaos. Basics of nonlinear time series analysis. Applications to chaos synchronization, chaos-based communication, chaos control, and neuromorphic computing. TIA: Systems and Controls.

Prerequisites

Math 2401 or 2411 or equivalent.

Math 2403 or 2413 or equivalent.

CEE/ISyE/MATH 3770 or ISyE 2027 or equivalent.

Textbooks

K.T. Alligood, T.D. Sauer, and J.A. Yorke, *Chaos. An introduction to dynamical systems*, Springer-Verlag, 1996. ISBN 0-387-94677-2 (required).

S.H. Strogatz, *Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and Engineering*, Second Edition, Westview Press, 2015. ISBN 978-0-8133-4910-7 (optional).

Instructor

Dr. Alexandre Locquet, Office 206

E-mail: alocquet@georgiatech-metz.fr. When you email me, please start the subject of the email with [ECE8803]. Failure to place this in the subject line could cause your email not to be read.

Lectures

Mondays , 10 am-11:30pm, yellow room

Wednesdays, 3:30pm-5pm, yellow room

Office hours

Wednesdays, 9am-10am, Office 206.

Homework

Problems will be assigned roughly once every two weeks. Please box all answers and staple all pages together. An assignment might be due on the week before final exams. You are allowed to collaborate on homework with other students, but all work to be submitted should then be worked out and written up on your own. Of course, copying solutions from a Faculty solutions manual is cheating.

Homework should be turned in at the beginning of a class. Homework turned in late loses 50%. Homework turned in two or more days late will not earn credit. Procedure for turning in **late** homework: drop it in my mailbox and send me an email (alocquet@georgiatech-metz.fr) as soon as you have done so.

Project

Students will choose a project related to a specific subject in nonlinear dynamics at the beginning of the term. Students are required to make a 5-minute presentation on January 31, a 10-minute presentation on February 28 and a final 15-minute presentation at the end of the term, as well as provide a 10-page report.

Examples of possible topics:

- Design of fuel efficient space missions (Lagrange points, Interplanetary Transport Network,...)
- Chaotic Mixing of microfluids
- Chaos synchronization of networks of dynamical systems (cluster formation, chimera states, etc...)
- Reservoir (neuromorphic) computing with nonlinear dynamical systems
- Heart nonlinear dynamics/ Sudden death forecasting
- Nonlinear dynamics in neuron behavior /inter-neuron communication
- Rogue waves
- Chaos in electronic circuits
- Chaos in lasers and synchronization of chaotic lasers
- Pattern formation in spatially extended systems
- Stochastic Resonance
- Stability/instability of electric power grids
- Nonlinear dynamics in gene regulation
- Chaos control
- Embedding theorems (nonlinear time series analysis) and their applications

Students will typically make extensive use of the digital library: <http://www.library.gatech.edu/> to get started on the project.

Assessment

Homework	20%
Project	50%
Final Exam	25%
Attendance	5%

The final examination will be closed-book and notes. It will be 3-hour long and comprehensive. The use of a calculator will be allowed. Any request for regrading a homework assignment must be made within one week of getting the homework back. Illness is an appropriate reason for missing the final examination, but you will need to produce a doctor's note stating that you are not able to take the final.

Students who do not miss more than 2 lectures during the entire term get 5% for attendance. Students who miss more than 2 classes, for any reason, get 0% for attendance. Attendance sheets will be used to count the number of absences.

Topical Outline

1. Dynamics of Iterated Maps

One- and Two-dimensional maps

Limit sets: fixed points, periodic points, chaotic attractors

Stability: sinks, sources, saddles, stable and unstable manifolds.

Chaotic Orbits: sensitivity to initial conditions, Lyapunov exponents, fractals.

2. Dynamics of Continuous-Time Systems

Linear and nonlinear systems. Poincaré-Bendixson theorem.

Limit sets: Equilibrium points, limit cycles, quasi-periodic attractors, chaotic attractors.

Stability: characteristic values, Lyapunov functions, Floquet multipliers, Lyapunov exponents.

Lab demonstrations: the Lorenz system, chaos in laser diodes.

3. Bifurcations

Saddle-node, period-doubling, Hopf, and torus bifurcations.

Bifurcation diagrams and routes to chaos: period-doubling cascade, quasi-periodicity, intermittency and crises.

Lab demonstration: period-doubling cascade in the Lorenz system.

4. Quantifying chaos

Lyapunov spectrum.

Fractal dimensions.

Kolmogorov-Sinai Entropy.

5. Basics of nonlinear time series analysis

State reconstruction from data through delay embedding.

Simple nonlinear prediction and noise reduction.

Computation of the largest Lyapunov exponent from data

6. Coupled chaotic systems and applications

Chaos control.

Synchronization of chaotic systems.

Chaos-based communications.

Optical communications using chaotic laser diodes.

Random-number generation

Neuromorphic computing

Compressive Sensing

Tentative Table of Contents

I. Basic Notions on Dynamical systems

I.1 What is Chaos?

I.2 Classification of Dynamical Systems.

I.2.1 What is a Dynamical System?

I.2.2 Classification

II. One-Dimensional Maps

II.1 Introduction and Definitions

II.2 Stability of Fixed Points

II.3 Periodic Orbits

II.4 Sensitive Dependence on Initial Conditions

II.5 Itineraries and Symbolic Dynamics

II.6 Bifurcations of Smooth One-Dimensional Maps

II.6.1 Period-Doubling Bifurcation

II.6.2 Saddle-Node Bifurcation

II.6.3 The Transcritical Bifurcation

III. Two-Dimensional Maps

III.1 Introduction and Definitions

III.2 Stability of Fixed Points

III.3 Linear Maps

III.4 Nonlinear Maps

III.4.1 Stability of Fixed and Periodic Points

III.4.2 Stable and Unstable Manifolds of Saddle Points

III.5 (Local) Bifurcations in 2D Maps

IV. Chaos

IV.1 Introduction

IV.2 Lyapunov Exponent of 1D Maps

IV.3 Chaotic Orbits

IV.4 Chaos in the Logistic Map $G=4x(1-x)$

IV.4.1 Conjugacy

IV.5 Lyapunov Exponents in Higher Dimensions

IV.6 Chaotic Orbits in Higher Dimensions

IV.7 Numerical Calculation of Lyapunov Exponents

IV.8 Chaotic Attractors

IV.8.1 Limit Sets

IV.8.2 Dimension of Chaotic Attractors

V. Continuous-Time Systems

V.1 Introduction

V.2 Existence and Uniqueness

V.3 Linear 1D Differential Equations

V.4 Equilibrium and Stability

V.5 Nonlinear 1D Differential Equations

v.6 n-Dimensional Linear Differential Equations

v.7 Stability Criterion for Linear and Nonlinear Systems

v.8 Nonlinear Differential Equations in More than One Dimension

V.8.1 Properties of forward-limit sets

V.8.2 The Poincaré-Bendixson Theorem

V.9 Chaos in Differential Equations

- V.9.1 Quasi-Periodic Signals
- v.9.2 Lyapunov Exponents
- V.9.3 Example of Chaotic Systems
- v.10 Bifurcation of Equilibrium Points
 - v.10.1 Saddle-Node Bifurcation
 - V.10.2 Andronov-Hopf Bifurcation
- V.11 Bifurcation of Limit Cycles
 - V.11.1 Saddle-Node Bifurcations of Limit Cycles
 - V.11.2 Period-Doubling Bifurcation of Limit Cycles
- V.12 Global Bifurcations and Crisis
- V.13 Famous Routes to Chaos
 - V.13.1 Period-Doubling Route to Chaos
 - V.13.2 Quasi-Periodic Route to Chaos
 - V.13.3 Intermittency Transition to a Chaotic Attractor
 - V.13.4 Transition to a Chaotic Attractor through a Crisis

VI. Experimental Time Series Analysis

- VI.1 Introduction
- VI.2 Embedding Theorems and Phase Space Reconstruction
- VI.3 Examples: modeling, prediction, parameter identification, estimation of attractor dimension

VII Applications of Chaos Theory

- VII.1 Chaos in Electronic Systems
- VII.2 Chaos in Laser Systems
- VII.3 Synchronization of Chaotic Systems and Chaos-Based Communication
- VII.4 Random-Number Generation
- VII.4 Neuromorphic Computing Exploiting Transient States in Dynamical Systems
- VII.5 Compressive sensing exploiting the pseudo-randomness of a chaotic system

Student-Faculty Expectations Agreement

At Georgia Tech we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body. See <http://www.catalog.gatech.edu/rules/22/> for an articulation of some basic expectation that you can have of me and that I have of you. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, I encourage you to remain committed to the ideals of Georgia Tech while in this class.

Honor Code

Students are, of course, expected to abide by the [Georgia Tech Honor Code](#) and the “GTL student policies” document that you will find in the resources section of the Tsquare website for this course. Instances of academic misconduct will be viewed very seriously. For any questions involving Academic Honor Code issues, please consult me or visit www.honor.gatech.edu.